ISSN (Online) 2456 -1304

International Journal of Science, Engineering and Management (IJSEM)

Vol 11, Issue 11, November 2024

Geometry of Perception: A Desargues the Connection of Cut-One

[1] Jonalee D. Divinagracia

[1] Iloilo Science and Technology University, Burgos St., La Paz, Iloilo City 5000 Philippines
Corresponding Author Email: [1] jodivinagracia@gmail.com

Abstract— - Both mathematicians and artists were curious about the guiding principles of this artistic method, and they started looking into the underlying logic of these rules. Artists were particularly interested in lines of sight and parallel lines that met and crossed at a vanishing point, and the mathematicians tried to find a system that would capture these ideas. As a result of this shared interest between artists and mathematicians, the Desargues theorem is just one of several early projective geometry theorems that address perspective. This study aimed to determine the projective geometry in contemporary arts through cut-one-plant two hundred paintings of an Ilonggo artist. Using the theoretical and document analysis, interviews, synopsis, and a critique of the work were made based on the photographs of the artwork. This paper outlines the connection of Renaissance art to contemporary art and discusses its mathematical implications and alignment of Cut-one given Desargues' geometry.

Index Terms: Cut-one, Desargues Theorem, Projective Geometry, Perspective Drawing.

I. INTRODUCTION

It is all the way you look at it. Art and mathematics share a prosperous historical relationship. The links between these two are apparent and inherent. One exists with the other. In art, when shapes are present, that is math. Math is when illusions provide one image from one point of view and different from another.

Paintings contain elements like structure, gesture, abstraction, narrative, and drawing, making them foms of visual art. The goal of painting in realism is to provide the viewer with an accurate representation of reality.

Cut-one-plant Two Hundred is an artwork by Joe Amora, an Ilonggo artist from San Enrique, Iloilo, Philippines, who has won several awards in painting and sculpture. His artwork will become the subject of the researcher to provide alignment in mathematics and arts given Desargues' geometry through linear perspective.

In contemporary painting, perspective has become more refined as a reflected component and a graphic tool. Desargues' theorem gracefully arises from the principles embedded in perspective painting. It is a result of projective geometry, among other things.

II. LITERATURE REVIEW

The following are key concepts in this study. They are discussed briefly to gain a shared understanding of what is being meant in the analysis of this study.

A. Perspective Drawing

The construction of linear perspective is one of the most significant innovations in drawing history. It depends on a set of rules and principles that assist artists in producing realistic depth in their drawings. One-point, two-point, and three-point perspectives are among the various selections of linear perspective (Sal, 2017).

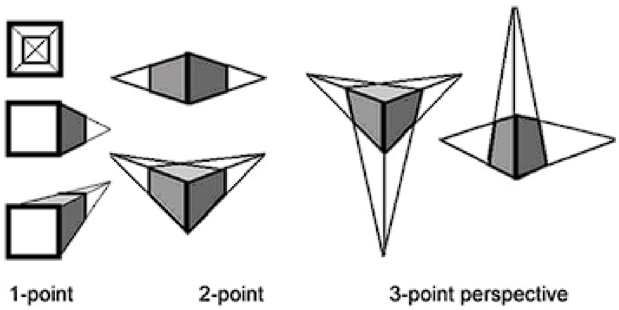


Figure 2.1a. Types of Linear Perspective Image credit from people.eecs.berkeley.edu

Over a century before Brunelleschi's perspective demonstrations, which popularised convergent perspective in the Renaissance, Giotto's "Jesus Before the Caïf" is recognised as one of the earliest paintings to utilise perspective (Science and Art of Perspective, 2004).

Figure 2.1b. "Jesus Before the Caif" by Giotto (1305) Image credit from ResearchGate

Through the technique of linear perspective, artists can create an illusion of depth in a flat scene by employing parallel lines that converge at a vanishing point on the horizon line. Perspective painting changes sizes and shapes. The rim of a cup may change from circular to elliptical, a structure in the distance might be smaller than a man's head, etc. However, lines always stay as they are, an application of geometry in art (Blasjo, V., 2009, Two Applications of Art to Geometry).

B. Girard Desargues (1591-1661)

Girard Desargues is the first mathematician acknowledged for his invention and contributions to projective geometry. Desargues's most notable contribution to perspective is the definition of points and lines to infinity. This concept, which provides the vanishing point more important than the result of geometric construction, is essential to projective geometry and plays a significant role in the evolution of perspective theory and practice (Baglioni et al.; R., 2022).

C. Projective Geometry

In mathematics, the relationship between the mappings and projections of actual three-dimensional objects onto a two-dimensional plane or piece of paper is called projective geometry. This type of geometry offers specific guidelines and instruments to convert a real-world view or object into a picture that makes it appear three-dimensional. This area of geometry can be seen in artistic forms such as painting and sketching (Early Use of Projective Geometry in Art, 2016).

Shadows cast by opaque objects and movies shown on a screen are examples of projections. Projective or perspective drawing (see Figure 2.3) occurs when the sight lines connecting the image in the reality plane (RP) to the artist's eye intersect the picture plane (PP). The horizon appears as a horizontal line parallel to PP (Artmann, B., 2018, Projective geometry).

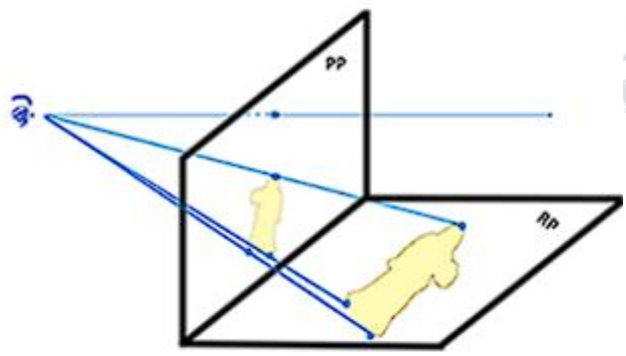


Figure 2.3. Projective Drawing

D. Perspectivity

Desargues' triangle theorem is a theorem in perspective geometry that deals with perspectivity. Consider a point O (see Figure 2.4) is not on distinct lines P and P'. If we correspond a point w, a, y on P from point O to a point w', a', y' on P', this one-to-one correspondence is known as perspectivity.

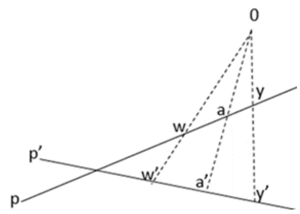


Figure 2.4

E. Projective Transformation

The points on P are said to be changed into the points of line P' by a perspective transformation with centre O. Now, if the points of P are changed to P' by a perspective transformation. Then, the points of P' are changed into points of another line P" (figure 2.5) by a perspective transfomation with another centre O'. If this remains an infinite number of times, the points of P will eventually correspond exclusively with points on the line P'', resulting in a relationship known as projectivity. The points of P have been changed into points of line P''… by a projective transformation (Moseley et al., 1968).

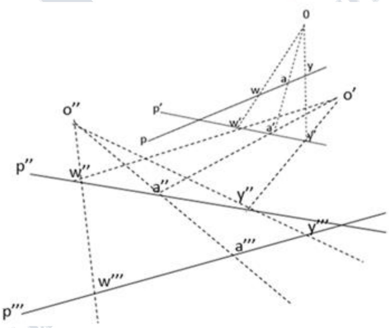


Figure 2.5

F. Desargues Theorem

Desargues' most important theorem, called Desargues' Theorem, involves two triangles in perspective. The theorem guarantees that parallel lines will intersect somewhere on the plane. Desargues' theorem states that the points of intersection of the corresponding sides of the two triangles, providing such intersection points are present, will all lie on the same line when two triangles are set so that the three lines linking their respective vertices come together at a single point. (Lange, M, 2015).

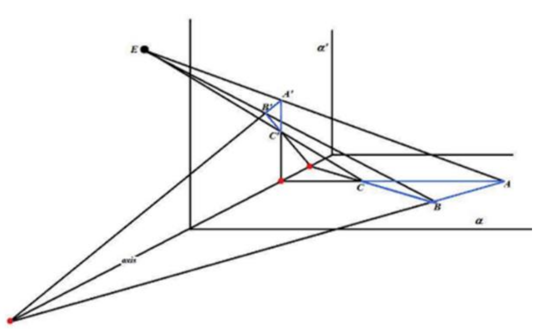


Figure 2.6a. An Illustration of Desargues' Theorem Image credit from www.semanticscholar.org

ISSN (Online) 2456 -1304
International Journal of Science, Engineering and Management (IJSEM)
Vol 11, Issue 11, November 2024

Triangle ABC is in the plane (see Figure 2.6a) a pyramid's base and triangle A'B'C' is located in the image plane, as well as the point where the lines (visual rays) joining comparable areas of the picture plane vertices intersect at E, which serves as the artist's focal point and the top of the visual pyramid. Notice that the two equivalent sides, A'B' and AB, are parallel. Desargues chooses "a point at infinity" to represent the intersection of these two parallel lines that meet somewhere on a plane (Smallwood, 2009).

Theorem: Assume that ABC and A'B'C' are two triangles with separate vertices (see Figure 2.5b) and that point O is where the lines AA', BB', and CC' intersect. Put differently, these triangles are in perspective concerning the rays of vision AA', BB', and CC' when viewed from the eye point O. Let X, Y, and Z denote the points where the lines AC and A'C', BC and B'C', and AB and A'B' intersect, respectively. Thus, X, Y, and Z are aligned on the same line (Markowsky, E., 2020).

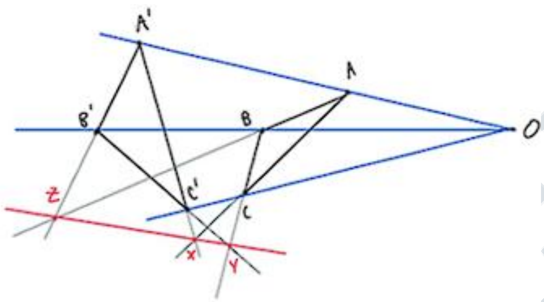


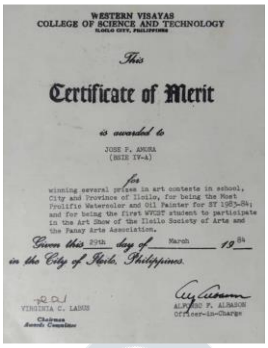
Figure 2.6b. Desargues' Theorem Proof Image credit from www.math.stonybrook.edu

G. Presentation of the artist and his artwork

Jose P. Amora

Known as Joe Amora (see Figure 2.7.1a) is a B. S. Industrial Education Graduate of the Iloilo Science and Technology University, formerly named WVCST. He received the Certificate of Merit (see Figure 2.7.1b) from his school as the first WVCST student who won several prizes in art contests in the school, City, and Province of Iloilo and participated in the art show of the Iloilo Society of the Arts and the Panay Arts Association in 1984.

(a)Joe Amora
An Ilonggo Artist and Sculpture Photo credit from Amora FB Account



WESTERN VISAYAS
COLLEGE OF SCIENCE AND TECHNOLOGY
ILOILO CITY, PHILIPPINES

This

Certificate of Merit

is awarded to

JOSE P. AMORA
(BSIE IV-A)

for

winning several prizes in art contests in school, City and Province of Iloilo, for being the Most Prolific Watercolor and Oil Painter for SY 1983-84; and for being the first WVCST student to participate in the Art Show of the Iloilo Society of Arts and the Panay Arts Association.

Given this 29th day of March 1984 in the City of Iloilo, Philippines.

VIRGINIA C. LADUS
Chairman
Awards Committee

ALFONSO F. ALBASON
Officer-in-Charge

(b) WVCST Certificate of Merit 1984
Photo credit from Joe Amora
Figure 2.7.1

As early as 1983, Amora joined Hubon Madiaas as a charter member. From a very illustrative style, he progressed, through experimentation, to conceptual works using indigenous materials, winning awards along the way. A true-blooded farm boy, Amora persistently worked on his art in his native hometown of San Enrique, where materials that abound in his place, such as vines and bamboo, became the main components of his artwork.

In addition, he joined the Arts Council of Iloilo Foundation, Inc. in 1986, followed by affiliation with the Visayas Island Visual Artists Association in 1990 and the Iloilo Visual Artists Association in 2000.

He has showcased his talent throughout his career through numerous solo and group exhibitions. Notable solo exhibitions include "Dingle: Yesterday, Now, and Tomorrow" at MCHS, Dingle, Iloilo, in 1986, and "Igang" Stone and Wood Sculpture at UPV Art Gallery, UPV Iloilo City, in 1996.

His participation in selected group exhibitions spans a wide range, from the Iloilo Society of Arts Grand Exhibition in 1983 to the CCP National Touring Exhibition, "Art and the Indigenous Elements," in Luzon, Visayas, and Mindanao in 1992. His collaborative efforts were evident in exhibitions like "Apat-Apat" with fellow artists Defensor, Orig, and Zoluaga at Hulot Aninipay, Museo Iloilo, and Iloilo City in 1992.

His artistic accomplishments have garnered multiple awards and honours, including recognition as a finalist in the slogan and poster competition held by the National Media Production Center Region VI in 1983. He secured third place in the Slogan and Poster Contest by the Iloilo Environmental Watchers Club in 1983 and second place in the NEA Painting Competition in 1986 and 1987.

Significantly, he secured first place in several competitions, including the Pinta-Paraw in 2000, 2001, and 2002 at Punta Villa, Arevalo, Iloilo City, as well as the Belen Making Contest at SM City, Iloilo City, in 2001.

His public art contribution includes being a foremost assistant sculptor for Ed Defensor in creating the "Lin-ay sang Iloilo" found at the top of the Iloilo City Hall Building in 2011.

His journey in the visual arts reflects his dedication, creativity, and profound impact on the cultural landscape of Iloilo and the Philippines.

Cut one, plant two hundred.

Cut-one, Plant Two Hundred, is an oil painting (see figure 2.7.2a), an entry of Joe Amora in Metrobank Art and Design Excellence in 1995 (see figure 2.7.2b). He used colour and linear perspectives to create a sense of depth in his painting. Every colour is symbolic; green represents youth, yellow represents hope, and brown represents maturity. His piece proves his artistic ability and commitment to raising public awareness of deforestation and acting as a catalyst for change. Both upland and lowland communities are crushed by deforestation, which leads to several issues. More trees must be planted to compensate for the losses and stop global warming.

(a) Cut one, Plant Two Hundred by Joe Amora (1995)
Image Credit from Amora FB Account

METROBANK FOUNDATION, INC.
presents this
CERTIFICATE
OF
PARTICIPATION
to
Jose P. Amora
for joining the
12TH METROBANK FOUNDATION
NATIONAL PAINTING COMPETITION
with the entry
"Cut One Plant Two...Hundred"
Given this 8th day of September, 1995
at the Metrobank Plaza, Makati.
HON. LUCRECIA R. KASILAG
GEORGE S.K. TY
PLACIDO L. MAPA, JR.

(b) Metrobank Inc. Cert. of Participation, 1995
Photo Credit from Joe Amora

Figure 2.7.2

III. CONCEPTUAL PARADIGM

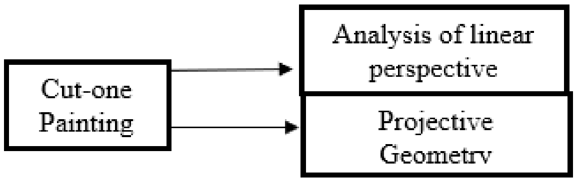


Figure 3.1. The relationship of the Cut-one painting, Linear perspective analysis, and projective geometry

The diagram shows the relationship between the cut-one painting, its linear perspective analysis, and its projective geometry. Using the linear perspective of the cut-one picture, the Desargues triangle theorem, which establishes the projective geometry, will be built.

IV. OBJECTIVES OF THE STUDY

This study aims to determine the projective geometry in cut-one painting.

Specifically, it aims to:

Illustrate Desargues' triangle theorem in cut-one linear perspective analysis.

V. METHODOLOGY

A. Research design

This research used theoretical and document analysis approaches. A critique of the painting was made based on the image of the artwork. The artists' background profiles and the history of making a painting were also described to understand more about the paintings' presentations. Since the interest of this study is more in the depth of analyses, only the qualitative approach was employed.

B. Subject of the Study

The study's subject is the cut-one, plant two hundred painting of an Ilonggo artist, Joe Amora.

C. Data Analysis

This study used the qualitative approach, which involves two levels of analysis—the first level analyses linear perspectives in Cut-one. Level two determines the projective geometry in Cut-one's painting.

D. Ethical Considerations

The study followed ethical standards, prioritising participants' well-being and maintaining anonymity. The researcher obtained consent for voluntary participation and used the information for the study's purpose. Six months after the final report, the researcher destroyed all hard copies and erased contents from a password-protected personal computer and backup drive. Ethical research promotes respectful dialogue between researchers and participants, aiming to transform experiences into transformative change.

ISSN (Online) 2456 -1304

International Journal of Science, Engineering and Management (IJSEM)

Vol 11, Issue 11, November 2024

Reinforcing dominant positions raises questions about the researcher's integrity and motivations, potentially marginalising and silencing the least advantaged (Creswell, 2005; Cacciattolo, 2014).

VI. RESULT AND DISCUSSION

A. The first level of Analysis

A. Cut-one Analysis of linear Perspective

According to Sal's study in 2017, one of the most critical developments in drawing history is the introduction of linear perspective. Its foundation consists of guidelines and precepts that help painters create genuine depth in their drawings.

Like Renaissance Artists, as a contemporary artist, Joe Amora also used one-point and two-point types of linear perspective in his painting (see Figure 6.1b). He used linear perspective to create an illusion of distance on a flat plane (see Figure 6.1a). Notice the effects on tones and colours that show perspective. It makes you feel how vast the fields are ahead. Allow your eyes to travel from a big tree to its roots that formed circular in one point dimension, to the number of trees, from the horizon line where the field meets the sky, to the objects in the distance that are visible to the observer's line of sight.

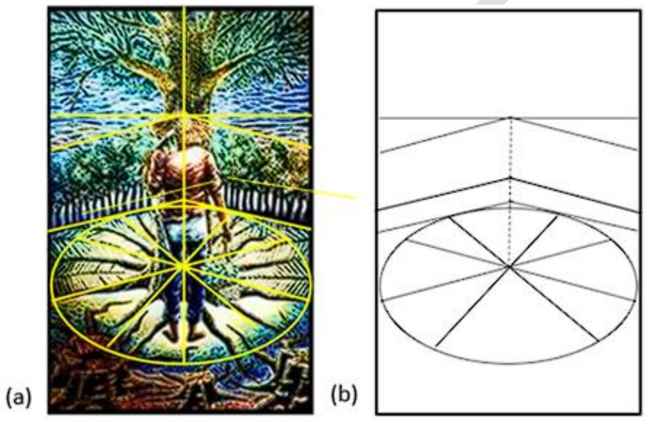


Figure 6.1. Cut-one Analysis of Linear Perspective Illustration

B. Projective Geometry seen in Cut-one

Girard Desargues, a French mathematician and engineer, became aware that another branch of geometry used only straight edges. Due to the nature of projective geometry, the fundamental theorem only uses straight edges and straight lines with no measurement involved.

Artists and designers use Desargues' theorem to demonstrate the geometric concepts underlying "photography" and "perspectivity," allowing them to portray three-dimensional objects on paper.

In the study of Baglioni and Migliari in 2022, Desargues is mainly known for having defined points and lines to infinity and for this contribution to perspective. This concept is essential to projective geometry and significantly impacts the evolution of perspective theory and application because it provides the vanishing point with a deeper meaning than the result of geometric construction.

In the study of Hilary Smallwood in 2009, Desargues' Theorem shows how two triangles drawn from perspectives relate to one another. The three intersections of pairs of matching sides are on a straight line if the corresponding vertices of two triangles are connected by three straight lines that meet at one point.

Theorem: Let the corresponding vertices of the pair of triangles APE and A'P'E', END and E'N'D', AIM and A'I'M', DOM and D'O'M' all lie on the same plane (refer to Figure 6.2). These lines meet at a single point, C. The corresponding sides AE and A'E', ED and E'D', DM and D'M', and AM and A'M meet at the intersection point.

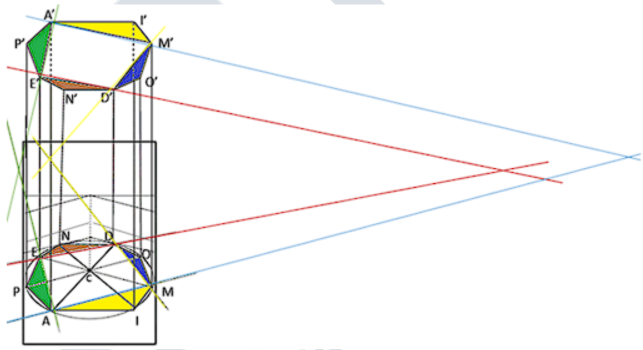


Figure 6.2. Desargues Linear Perspective Construction of Cut-one with Vanishing point

Remember that each line goes forever beyond the segment that makes up a side of either triangle in perspective. Unless AI and A'I' are parallel, they will intersect at some point on the plane.

The Desargues theorem hides a significantly more captivating geometric structure behind its presentation. Currently, this arrangement is known as the Desargues configuration. In geometry, the Desargues configuration (configuration) consists of ten points and ten lines, each containing three points and associated with three lines.

According to Markowsky's 2020 study, Desargues's theorem, which involves two triangles in perspective, is stated as follows.

Theorem: Let CUT and C'U'T' be two triangles with distinct vertices, such that the lines

CC', UU', and TT' are incidents in point V. In other words, these triangles are in perspective from the eye point V, with rays of vision CC', UU', and TT'. Let the points of intersection of the pair of lines, CU and C'U', be denoted by O (see figure 6.3), the pair CT and C'T' by N, and the pair UT and U'T' by E. Then O, N, and E are collinear.

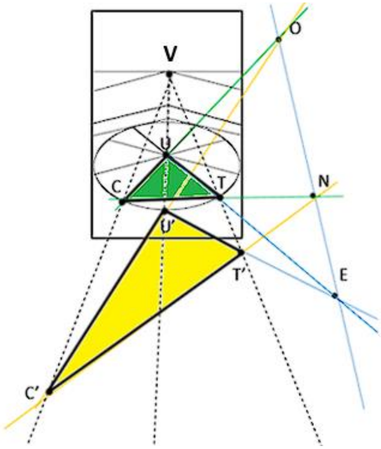


Figure 6.3. Illustration of Desargues Configuration (10_3) of Cut-one

Proof: To generate the image above, begin with an eye point V, enclosed in at least three arbitrary planes. Each of these three planes will contain a line, which we will call CC', UU', and TT', respectively. These are characterised by the broken lines in Fig. 6.3 and can be considered to represent rays of vision. Imagine that the points V, C, U, and T have been given some height, while all other points remain in the image plane P. Then the points C', U', and T' lie on some other plane P', and can be thought of as a projection of CUT from the plane P onto P'. The planes P and P' will meet in a line ℓ, which follows from the second axioms of a projective plane, since any two lines P_1 in P and P_2 in P' must be incident at a point. Recall our initial three planes containing the rays of vision CC' UU' and TT'. The four points C, C', U, and U' will lie on one of these planes. Thus, the intersection of this plane with planes P and P' will yield the point O. The points N and E are similarly constructed when we repeat this process for the sets of points C, C', T, T', and U, U', T, T'. The points O, N, and E will all lie on the line ℓ, indicated by the blue line in Fig. 6.3 Thus, we have proved Desargues' Theorem where the triangles lie in some three-dimensional object space.

Theorem: Two triangles are perspective relative to a point if and only if they are relative to a line. (Pamfilos, P., 2019).

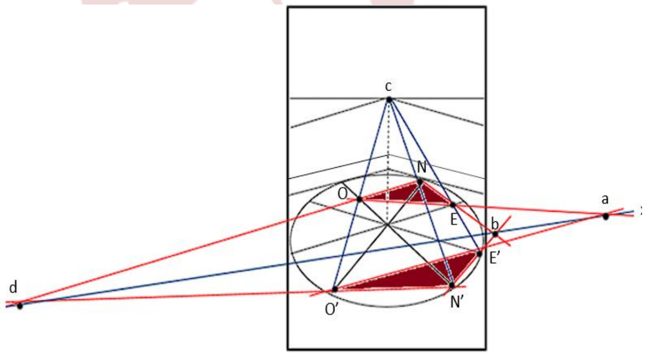


Figure 6.4. Three-Dimensional Desargues Theorem proof of Cut-one

Two triangles are called "perspective concerning a point" or "point perspective" when we can name them ONE and O'N'E' in such a way that lines {OO', NN', EE'} pass through a common point c. Points {O, O'} are then called "homologous", and similarly, points {N, N'} and {E, E'}. The point c is called the "perspectivity centre" of the two triangles. The two triangles are called "perspective concerning a line" or "line perspective" when we can name them ONE and O'N'E' in such a way (See Figure 6.4) that the points of intersection of their sides d = (ON, O'N'), b = (NE, N'E') and a = (E O, E'O') are contained in the same line x. Sides ON and O'N' are then called "homologous", and similarly, the side pairs (NE, N'E') and (EO, E'O'). Line x is called the "perspectivity axis" of the two triangles.

According to Marc Lange's study in 2015, Desargues' theorem states that two triangles are in perspective. It is expressed as follows: When two triangles are set so that the three lines connecting their respective vertices come together at a single point, the points of intersection of the corresponding sides of the two triangles, assuming such points exist, also lie on a single line.

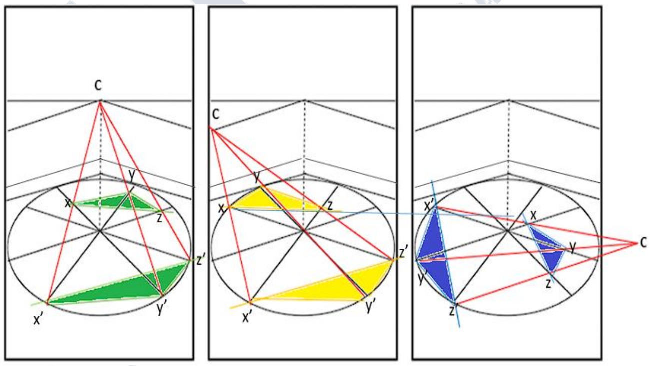


Figure 6.5. Illustration of Desargues' Theorem proof of cut-one in different Vanishing point

Desargues theorem explains that each line in perspective extends infinitely beyond the section that makes up a side of one of the two triangles. The pair of triangles xyz and x'y'z' on the plane (see Figure 6,5) where lines xx', yy', zz' lie on a single line and intersect at point c. Then, a pair of corresponding sides xy, yz, zx and x'y', y'z', z'x' meet at the intersection point.

VII. CONCLUSIONS

We have seen how the artistic techniques of Renaissance-inspired art are used today. The use of linear perspectives found in Cut-one paintings shows the connection between Renaissance arts and contemporary art. Linear perspective analysis in Cut-one paintings becomes the basis for Desargues's triangle theorem construction.

Desargues triangle theorem proved the alignment of the Cut-one in views of Desargues geometry, then the projective geometry in the Cut-one painting was determined as given proof above:

a. The Cut-one Desargues' linear perspective construction,
b. The Cut-one illustration of Desargues' theorem at various vanishing points
c. The Cut-one Desargues Configuration and
d. The Cut-one three-dimensional viewpoints.

VIII. RECOMMENDATIONS

School. This study will help the institution attract staff and students by making it more competitive and contributing to accreditations.

Teachers. This research will help teachers, especially those specialising in mathematics and art, provide additional knowledge.

Students. This research will provide insights to students who study in the same area, enabling them to understand better and learn.

IX. ACKNOWLEDGEMENT

I would love to express my gratitude to Joe Amora for his participation in meeting this journal's needs. In addition, I would like to thank my family for their unwavering support, my professors Dr. Therine Yap and Dr. Saul Muyco for their help, and most importantly, Almighty God for giving me the knowledge and fortitude I need.

*The author declared no conflict of interest in publishing this article.

REFERENCES

[1] Architecture, S. (2017, November 18). Perspective drawing research. SS Archi. https://55archi.wordpress.com/2017/11/15/1697/
[2] Artmann, B. (1999, July 26). Projective geometry | Points, Lines & Planes. Encyclopedia Britannica. https://www.britannica.com/science/projective-geometry
[3] Baglioni, L., & Migliari, R. (2022). Desargues's Perspective Theory: A critical interpretation of the Fundamental Theorem. Nexus Network Journal, 25(3), 495–520. https://doi.org/10.1007/s00004-022-00636-6
[4] Blasjo, V. (2009). Two Applications of Art to Geometry. The Montana Mathematics Enthusiast, ISSN 1551–3440, Vol. 6, No. 3, pp. 297–304.
[5] Cacciattolo, M (2014). Ethical Considerations in Research. The Praxis of English Language Teaching and Learning. Critical New Literacies pp. 61-79. DOI:10.1007/978-94-6300-112-0_4
[6] Creswell, J.W. (2013). Research Design: Qualitative, Quantitative, and Mixed Methods Approaches. New York, NY: SAGE Publications, Inc.
[7] Early use of projective geometry in art. (n.d.). https://www.ijser.org/paper/Early-Use-of-Projective-Geometry-in-Art.html
[8] Human verification. (n.d.). https://www.semanticscholar.org/paper/Projective-Geometry%3A-Perspectives-from-Art-and-Smallwood/0dfabe1d7ab9b42605b3c64e00836ceb28ff276f
[9] Lange, M. (2015). Explanation, Existence and Natural Properties in Mathematics - A Case Study: Desargues' Theorem. Dialectica, 69(4), 435–472. https://doi.org/10.1111/1746-8361.12120
[10] Markowsky, E. (2020). Projective Geometry Seen in Renaissance Art. Stony Brook Department of Mathematics. http://www.math.stonybrook.edu
[11] Moseley, A. C. (1968). Some Proofs of Desargues' Triangle Theorem. Retrieved from https://digitalcommons.pvamu.edu/pvamu-theses/1301
[12] Pamfilos, P. (2019). Desargues' theorem and perspectivities. http://users.math.uoc.gr/~pamfilos/eGallery/problems/Desargues.pdf
[13] Perspective: The role of perspective in shaping the Renaissance. (n.d.). https://www.webexhibits.org/sciartperspective/perspective1.html
[14] Interview with Joe Amora, November 2023.